

Isotopic ratio, isotonic ratio, isobaric ratio and Shannon information uncertainty*

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(Dated: September 12, 2014)

The isoscaling and the isobaric yield ratio difference (IBD) probes, which both are constructed by yield ratio of fragment, provide cancelation of parameters. The information entropy theory is introduced to explain the physical meaning of the isoscaling and IBD probes. The similarity between the isoscaling and IBD results is found, i.e., the information uncertainty determined by the IBD method equals to $\beta - \alpha$ determined by the isoscaling [α (β) is the parameter fitted from the isotopic (isotonic) yield ratio].

PACS numbers: 25.70.Pq, 25.70.-z, 25.70.Mn, 89.70.Cf

Keywords: Shannon information, isotopic ratio, isotonic ratio, isobaric ratio

I. INTRODUCTION

In heavy-ion collisions (HICs) above intermediate energy, nuclear matters from sub-saturation to supra-saturation densities can be produced, which makes HICs be a unique experimental method to study the abnormal nuclear matters on earth. The supra-saturation nuclear matter, which is produced in the compression of the overlapping zone of the projectile and target nuclei, can not be probed directly. The supra-saturation nuclear matters produced in HICs are related to dense nuclear matters in compact astronomical body like neutron star, which attracts much interest both theoretically and experimentally. On the theoretical side of studying the HICs, the descriptions of the compressing and expanding of the collisions, at the same time the decay of the hot fragments still face many challenges. On the experimental side, the whole processes of the reaction like a black-box, with only the emitted light particles and final residues measurable. Most of the probes detecting the processes of HICs are based on the measurable light particles or final fragments [1]. But the final fragments carry only part information of the initial collisions since they undergo the decay process. Depending on the density and temperature, the nuclear symmetry energy is one of the important properties of nuclear matter. Since nuclear symmetry energy can not be measured directly, the many results of nuclear symmetry energy, which are extracted based on different indirect probes, are in conflict. Till now, the nuclear symmetry energy is still an open question in nuclear physics, and it is still important to find new probes to study the nuclear symmetry energy [1–3].

The isoscaling method, which uses the isotopic or isotonic yield ratio, is one of the important methods to study the nuclear symmetry energy of the sub-saturation

nuclear matter produced in HICs [4–6]. The isobaric ratio methods, which use the isobaric yield ratios, have been proposed to study the nuclear symmetry energy of finite nuclei [7–17], the chemical potential difference between neutrons and protons [18, 19], and the density difference between projectiles [20]. The volume effects manifested in the results [21] are found to originated from the neutron-skin of neutron-rich fragments [22, 23]. Besides, the ratios of fragments are also used to detect the temperature of the reaction [24–28]. A systematic comparison between the results of the isoscaling and the isobaric yield ratio difference (IBD) methods proves that the results of isoscaling and IBD are similar [18, 19]. In both the isoscaling and IBD methods, the yield ratios of fragments provide cancelations of special terms or parameters influencing the yield of fragment, which facilitates the study of nuclear symmetry energy [11–13].

The Shannon information theory is a method to measure the uncertainty in a random variable which quantifies the expected value of the information contained in a message, and can extract reliable information in the information transition from measured observable [29–32]. The Shannon information theory has many similarity compared to the black-box characteristics of the HICs processes. The ideas of Shannon information entropy has been introduced to study the hadron decaying branching process [33], and probe the liquid-gas transition in the disassemble of the colliding system in HICs [34]. In this article, we will introduce the information entropy theory to understand the isoscaling and IBD probes.

II. SHANNON INFORMATION ENTROPY THEORY

In the Shannon information theory, considering a system which has multi events $S = \{e_1, e_2, \dots, e_n\}$ with the corresponding probability $\{p_1, p_2, \dots, p_n\}$, the information uncertainty of a certain event e_i (or the information e_i contained) is defined as,

$$U(e_i) = -\ln p_i, \quad (1)$$

with $U(e_i)$ in units of *nats*.

* Supported by National Natural Science Foundation of China under Grant No. 10905017, Program for Science & Technology Innovation Talents in Universities of Henan Province (13HASTIT046), and the Young Teacher Project in Henan Normal University.

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If the probability of the event is non-uniform, the information entropy of the system can be defined as [29, 31, 33, 34],

$$H(S) = - \sum_{i=1}^n p_i \ln p_i. \quad (2)$$

The information entropy and information uncertainty can be used interchangeably [31]. In some applications, $U(e_i)$ is also named as information entropy of one event. To differ the concept of the previously defined information entropy in Refs. [33, 34], $U(e_i)$ is called as the *information uncertainty*. In HICs, all the types of particles and fragments form a system. Each type of particle and fragment can be viewed as an independent event with different probability denoted by yield or cross section (σ). In the work of Y. G. Ma, the liquid-gas transition is found in the information entropy of the system [34]. In this work, we concentrate on the information uncertainty of the final fragments, which is believed to carry part of the information of reactions.

III. ISOTOPIC/ISOTONIC/ISOBARIC RATIO AND INFORMATION UNCERTAINTY

In the free energy based theories describing the HICs above the Fermi energy, the yield of a fragment follow the exponential function, which is mainly decided by the free energy, the chemical properties of the source, and temperature [7, 11, 35–38]. In the ratio of fragment yield, the cancelation of some terms makes it possible to study the retained terms in the exponential function, and specific physical parameters can be studied. Some probes based on yield ratios are proposed, for examples, the isotopic temperature probes [24, 25] and the isobaric temperature probes [26–28], the isoscaling probe for nuclear symmetry energy [4, 5], and the isobaric yield ratio difference probes for nuclear symmetry energy [11, 13, 18–20], etc. We will explain the physical meaning of the result for the isoscaling and isobaric ratios using the information uncertainty theory.

Assuming the thermal equilibrium, in the grand-canonical ensembles theory within the grand-canonical limit, the yield of a fragment is given by [37, 38],

$$\sigma(A, I) = CA^\tau \exp\{[F(A, I) + \mu_n N + \mu_p Z]/T\}, \quad (3)$$

where A and $I = (N - Z)$ denote the mass and neutron-excess of the fragment; C depends on the reaction system; $F(A, I)$ is free energy of the fragment, and μ_n (μ_p) denotes the chemical potential of the neutrons (protons). In the modified Fisher model, the yield of a fragment is described in a similar form by considering the entropy of exchanging the neutrons and protons [11, 35]. The exponential law of the fragment yield makes it easy to explain the probes based on the fragment yield by using the information uncertainty theory since in Eq. (1) the logarithm operation of yield probability frees the parameters in the yields in Eq. (3).

A. Isotopic & Isotonic Ratios

For the isoscaling method, which uses the isotopic ratio and isotonic ratios, we denote a fragment with neutron numbers N and proton numbers Z as (N, Z) for convenience. From Eq. (3), which represents the residue probability for a fragment, the information uncertainty included in the fragment can be written as,

$$U(e) = -\ln\sigma(N, Z), \quad (4)$$

with e denoting an event corresponding to the fragment (N, Z) . In reactions of similar measurements, the temperatures of the specific fragment can be assumed as the same, thus the free energies of (N, Z) equal in the two reactions. The difference between the information uncertainty of fragment (N, Z) in the two reactions can be written as,

$$\Delta_{21}U(e) = U_2(e) - U_1(e) = \ln\sigma_1(N, Z) - \ln\sigma_2(N, Z), \quad (5)$$

Inserting Eq. (3) into (5), one obtains,

$$\begin{aligned} \Delta_{21}U(e) &= c1 - c2 + [F_1(N, Z) - F_2(N, Z) + N(\mu_{n1} - \mu_{n2}) \\ &\quad + Z(\mu_{p1} - \mu_{p2})]/T \\ &= \Delta c + [\Delta F_{12}(N, Z) + N\Delta\mu_{n12} + Z\Delta\mu_{p12}]/T, \end{aligned} \quad (6)$$

$\Delta c = c1 - c2$, with 1 and 2 are indexes denoting the reactions and the corresponding parameters. Defining $\alpha = \Delta\mu_{n21}/T = (\mu_{n2} - \mu_{n1})/T$ and $\beta = \Delta\mu_{p21}/T = (\mu_{p2} - \mu_{p1})/T$, Eq. (6) can be written as,

$$\Delta_{21}U(e) = \Delta c - N\alpha - Z\beta \quad (7)$$

with α (β) being the fitting parameter from the isotopic (isotonic) ratio between reactions. For isotopic ratio, $Z\beta$ is a constant (labeled as C_z). From Eq. (7), the following can be obtained,

$$\Delta_{21}U(e_p) = \Delta c + C_z + N\Delta\mu_{n12}/T. \quad (8)$$

Similarly, for isotonic ratios, $N\alpha$ is a constant (labeled as C_n). From Eq. (7), the following can be obtained,

$$\Delta_{21}U(e_n) = \Delta c + C_n + Z\Delta\mu_{p12}/T, \quad (9)$$

with e_p and e_n denoting the isotopic and isotonic events. It is shown that $\Delta_{21}U(e_n)$ depends on the reaction systems due to Δc_{12} . But only α and β are the interested parameters. For isotopic (isotonic) ratio, the C_z (C_n) is assumed to be a constant. This assumption can only be fulfilled when the nuclear density does not change.

One fragment belongs both to an isotopic chain and an isotonic chain. In the isoscaling analysis, the fragment is related to α in its isotopic ratio and to β in its isotonic ratio simultaneously. The difference between the information uncertainty included in a fragment from its isotopic ratio and isotonic ratio is,

$$\Delta_{12}U(e_p) - \Delta_{12}U(e_n) = N\alpha - Z\beta + C_z - C_n = 0. \quad (10)$$

If $C_z = C_n$ can be fulfilled, one has,

$$\frac{\alpha}{\beta} = \frac{Z}{N}. \quad (11)$$

This can only happen in the neutron-proton symmetric matter.

B. Isobaric Ratio

For isobaric ratio, the fragment will be denoted as (A, I) . The information uncertainty difference between the isobars differing 2 in I can be written as,

$$\Delta U(e_b)_{(I+2, I)} = \ln \sigma(A, I) - \ln \sigma(A, I+2), \quad (12)$$

where e_b denotes the isobaric event. Inserting Eq. (3) into Eq. (12), the CA^τ term cancels out and one obtains,

$$\Delta U(e_b)_{(I+2, I)} = [\Delta F(A, I, I+2) - \mu_n + \mu_p]/T, \quad (13)$$

with $\Delta F(A, I, I+2) = F(A, I) - F(A, I+2)$. Assuming the temperatures of two reactions are the same, one can define the difference between the information uncertainty of isobars,

$$\Delta_{21} U(e_b)_{(I+2, I)} = [\Delta_{21} F(A, I, I+2) - \Delta \mu_{n21} + \Delta \mu_{p21}]/T, \quad (14)$$

$\Delta_{21} F(A, I, I+2) = 0$ can be assumed, which results in the following equation,

$$\Delta_{21} U(e_b)_{(I+2, I)} = (-\Delta \mu_{n21} + \Delta \mu_{p21})/T = \beta - \alpha, \quad (15)$$

$\Delta \mu_{n21}$ ($\Delta \mu_{p21}$) is the same as in Eq. (6), which denotes the difference between the chemical potential of neutrons (protons) of the two reactions. In Eq. (15), the correlation between the isobaric ratio difference and the isoscaling parameters α and β is explicated. This correlation has also been illustrated and verified experimentally [18, 19].

C. Discussion

Both the isoscaling and isobaric methods use the yields of fragments produced in two similar reactions. In the

isoscaling and IBD methods, the free energies of the fragments cancel out in different manners by assuming the same temperatures of the reactions. In the isotopic (isotonic) ratios, the constant $\Delta_{12}c$ in Eqs. (8) and (9) makes the difference between the information uncertainty of the isotopic (isotonic) ratios depends on the reaction system, but it is unimportant in the isoscaling analysis since it only cares α and β . In the isobaric ratio, the cancelation of CA^τ makes it convenient to compare the fragment yield in reactions besides those induced by isotopic projectiles or on isotopic targets [13, 18, 19]. In the real reactions, the yield of fragment sometimes does not obey the isoscaling, in which case the isoscaling analysis encounters difficulties. The isobaric ratio, which uses only two or three isobars, does not require regular distributions of fragments as in the isoscaling method.

When comparing the information uncertainty included in the isoscaling and IBD probes, it should also be pointed out that the IBD results are obtained directly from the fragment ratio, while the isoscaling results are obtained indirectly since α and β are the fitting parameters from the isotopic or isotonic ratio. From the information theory, the IBD probe has advantages compared with isoscaling, and it should be more sensitive to the change of the reactions.

IV. SUMMARY

The information entropy theory is introduced to explain the isoscaling and IBD probes. The physical meanings of the isoscaling and IBD probes, which both use fragment ratio to make cancelation of parameters, are explained in the information uncertainty manner. The similarity between the isoscaling and IBD results is found, i.e., the information uncertainty determined by the IBD method equals to the value of $\beta - \alpha$. The IBD probe is shown to have advantage to the isoscaling method both in experiment and theoretical analysis, which could also be used when the fragment does not obey the isoscaling.

Acknowledgment

We thank Prof. Ma Yugang, at Shanghai Institute of Applied Physics, Chinese Academy of Sciences), for the useful discussion of the information entropy theory.

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